tance d of the point O' on the x axis, where $M = 0.3$, by the empirical relation

$$\delta^*/d \approx u_2/u_2' \tag{7}$$

The sonic velocity c at O' is approximately equal to $(kgRT_2)^{1/2}$, and the Mach number is

$$M = 0.3 = u_2'/c \tag{8}$$

Using Eq. (8), the velocity u_2' is obtained, and in Eq. (7), δ^* is known and the velocity u_2 (downstream of the shock) is obtained from normal-shock relationships.[3] Thus the point O' is established by the distance d of Eq. (7). For $M = 0.3$, u_2' is equal to u_θ of Eq. (5). By use of Eq. (5), angle θ is found. Thus point $B(x,y)$ on the body is established.

In Eq. (6), the values of X and Y are known for point B. Thus the radius r of the circle through points B,O', and B', and with the center on the x axis, is found from Eq. (6).

The arc $BO'B'$ is the required line of $M = 0.3$.

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Correlation of Hypersonic Static-Stability Data from Blunt Slender Cones

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CORRELATIONS of experimental hypersonic static-stability data from blunted slender cones have been obtained using simple Newtonian theory.[1] The basis for the correlation is developed in Ref. 1, and the purpose of this note is to present hypersonic static-stability data from blunt slender cones in a correlated manner suitable for use in obtaining quick, reasonably accurate predictions.

The nomenclature used is noted in Fig. 1, and the correlations of normal-force coefficients and pitching-moment coefficients are presented in Figs. 2 and 3, respectively. The correlations are based on the parametric dependence developed in Ref. 1, that is,

$$C_N \propto \alpha[2 + (\alpha/\theta_C)](1 - \xi^2)$$

and

$$C_m \propto C_N\{(2/3\theta_C)[(1 - \xi^3)/(1 - \xi^2)] - \xi[(1 - \theta_C)/\theta_C]\}$$

The correlations contain experimental data[1,2] covering a Mach number range from 8 to 22 and a bluntness ratio range from 0 (sharp) to 0.5. The Mach number "independence" of these essentially inviscid data is evident. Apparently for these cases Mach number 8 is sufficiently high to establish the limiting hypersonic static stability for these simple shapes.

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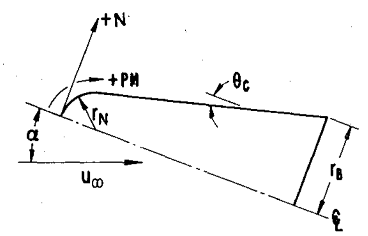

Fig. 1 Cone nomenclature

$$C_N = \frac{N}{(1/2)\,\rho_\infty U_\infty^2 S_B}$$

$$C_m = \frac{PM}{(1/2)\,\rho_\infty U_\infty^2 S_B d_B}$$

$$\xi = r_N/r_B$$

$$S_B = \pi r_B^2$$

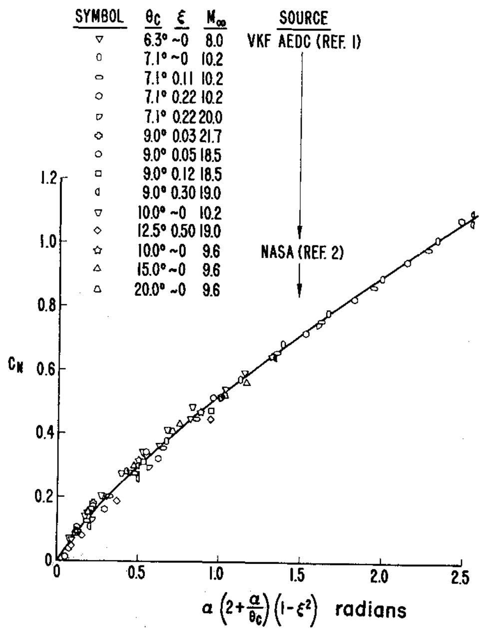

Fig. 2 Correlation of normal-force coefficients from blunt slender cones

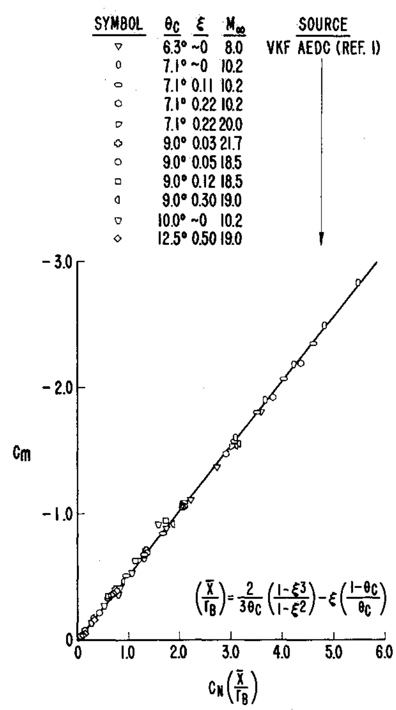

Fig. 3 Correlation of pitching-moment coefficients from blunt slender cones

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Radial Vibrations of Thick-Walled Orthotropic Cylinders

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In the design of modern missiles and space vehicles, increasing use is being made of the newer materials, such as reinforced plastics, whisker materials, and fiber-reinforced metals. These materials are essentially elastically orthotropic; that is, the Young's modulus of elasticity and Poisson's ratio differ in the three mutually perpendicular directions. Approximate formulas are developed here for the natural wavelengths associated with free radial vibrations of a thick-walled, infinitely long, orthotropic cylinder. The method has been employed previously by McFadden[1] for the case of isotropic cylinders.

For purely radial vibrations, the particle displacement $u(r,t)$ is governed by the equation

$$c_{11}(\partial^2 u/\partial r^2) + (c_{11}/r)(\partial u/\partial r) - c_{22}(u/r^2) = \rho(\partial^2 u/\partial t^2) \quad (1)$$

where r is the radial coordinate, t the time, and

$$c_{11} = \eta E_r(1 - \nu_{\theta z}\nu_{z\theta})$$

$$c_{22} = \eta E_\theta(1 - \nu_{zr}\nu_{rz})$$

$$1/\eta = 1 - \nu_{\theta r}\nu_{r\theta} - \nu_{rz}\nu_{zr} - \nu_{z\theta}\nu_{\theta z} - \nu_{rz}\nu_{z\theta}\nu_{\theta r} - \nu_{zr}\nu_{r\theta}\nu_{\theta z} \quad (2)$$

Note that, as opposed to the two elastic constants E and ν for an isotropic material, nine elastic constants E_r, E_θ, $\nu_{r\theta}$, etc., are required to describe the behavior of an orthotropic material.

If one assumes $u(r,t)$ in the form

$$u(r,t) = U(r)e^{i\omega t} \quad (3)$$

then $U(r)$ satisfies the equation

$$r^2(d^2U/dr^2) + r(dU/dr) + (k^2r^2 - n^2)U = 0 \quad (4)$$

where

$k = \omega/C_c$ is a wave number
ω = angular frequency
$C_c = (c_{11}/\rho)^{1/2}$ is the phase velocity of compressional waves
$n^2 = c_{22}/c_{11}$

Equation (4) is the familiar Bessel's equation of order n and argument kr. The general solution is

$$U(r) = AJ_n(kr) + BY_n(kr) \quad (5)$$

where A and B are constants of integration.

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[1] McFadden, J. A., "Radial vibrations of thick walled hollow cylinders," J. Acoust. Soc. Am. 26 (September 1954).

For free motions, the radial stress σ_{rr} vanishes on the inner and outer surfaces $r = a,b$, respectively. Hence the boundary conditions in terms of the displacement are

$$c_{11}(dU/dr) + c_{12}(U/r) = 0 \text{ on } r = a,b \quad (6)$$

where

$$c_{12} = \eta E_\theta(\nu_{r\theta} + \nu_{rz}\nu_{z\theta})$$

These boundary conditions result in the frequency equation given by

$$\begin{vmatrix} ka\,J_{n-1}(ka) - \beta\,J_n(ka) & ka\,Y_{n-1}(ka) - \beta\,Y_n(ka) \\ kb\,J_{n-1}(kb) - \beta\,J_n(kb) & kb\,Y_{n-1}(kb) - \beta\,Y_n(kb) \end{vmatrix} = 0 \quad (7)$$

with $\beta = n - c_{12}/c_{11}$.

Solution of Frequency Equation for Extensional Mode

Consider first the extensional mode. The frequency equation may be written as

$$F(ka) = F(kb) \quad (8)$$

where

$$F(x) = \frac{x\,J_{n-1}(x) - \beta\,J_n(x)}{x\,Y_{n-1}(x) - \beta\,Y_n(x)} \quad (9)$$

This function $F(x)$ is zero at $x = 0$ and increases with x until a maximum is reached at $x = x_0$ given by

$$x_0 = [n^2 - (n - \beta)^2]^{1/2} \quad (10)$$

From here, the function decreases with x approaching $-\infty$ and then begins at $+\infty$ and decreases, etc. The function $F(x)$ may be developed now in a Taylor series about the point x_0 to yield the following expansion:

$$F(x) = F(x_0) + \frac{(x - x_0)}{1!}F'(x_0) + \frac{(x - x_0)^2}{2!}F''(x_0) + \ldots \quad (11)$$

If one employs this expansion in the frequency equation [Eq. (8)] and writes

$$b = a(1 + \delta) \qquad ka = x_0\sum_{s=0}^{\infty} a_s\,\delta^s \quad (12)$$

the constants a_s are obtained by solving successively equations of higher order in δ, and the resulting wavelength $\lambda = 2\pi/k$ is

$$\lambda = (2\pi a/x_0)[1 + (\delta/2) + (\tfrac{1}{8})(m - 1)\delta^2 - (\tfrac{1}{16})(m - 1)\delta^3 + 0(\delta^4)] \quad (13)$$

where

$$m = \tfrac{2}{3} - (\tfrac{4}{3})(n - \beta)$$

The approximate expression (13) for λ may be identified with the mth-order mean radius and written as

$$\lambda \simeq (2\pi/x_0)[(a^m + b^m)/2]^{1/m} + 0(\delta^4) \quad (14)$$

Note that for an isotropic material

$$n = 1 \qquad \beta = (1 - 2\nu)/(1 - \nu) \qquad m = \frac{2(1 - 3\nu)}{3(1 - \nu)}$$

and the result reduces to that developed by McFadden.

Thickness Modes

Consider now the thickness modes of a hollow orthotropic cylinder. The Bessel functions in Eq. (7) may be replaced by their asymptotic expansions with the result

$$\tan k(b - a) \sim (b - a)[8\beta + 4(n - 1)^2 - 1]/(8kab) \quad (15)$$